Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

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MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO JUNE 14 EXAM, 2013

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In a dynamic flex-price economy with a cash-in-advance constraint on consumption and investment purchases, the Friedman rule is optimal.

A TRUE In such a setting, the CIA constraint binds when the nominal interest rate is positive. In that case, the intertemporal savings decision is distorted, implying too little investment in real capital. This inefficiency arising from the CIA constrain can be eliminated precisely with a zero nominal interest rate; i.e., when the Friedman rule is implemented.

(ii) In the basic New-Keynesian model, the optimal policy response to an inflation shock (i.e., a "cost-push" shock) is time inconsistent.

A TRUE An inflation shock will in this model create a trade off for the monetary policymaker. Reducing inflation by contractive policy has a cost in terms of a lower output gap. Optimally, the policymaker can reduce the costs by affecting inflation expectations downwards. I.e. by committing to a future contrative policy. This, however, is not time consistent, as it will *not* be optimal to conduct contractive policy when the future arrives and the inflation shock is gone.

(iii) In the Auerbach and Obstfeld (2005, *American Economic Review*) model, a temporary change in the money supply does not affect prices when the zero lower bound on the nominal interest rate binds.

A TRUE When the zero lower bound binds, money and bonds are perfect substitutes. All things equal an increase in the money supply will be fully absorbed, with no effect on prices. In their model, there is a fixed date after which the zero lower bound does not bind, and after which increases in the money supply will increase prices. Since price (and wage) determination is forward looking, current prices will only be affected by changes in the money supply if the change affects the money supply *after* the zero lower bound does no longer bind. A temporary change will not do this by definition, and will therefore have no effect on prices. It could be mentioned that a change, which is expected to change the future money supply (e.g., in the case of base drift), will have effect on current prices.

QUESTION 2:

Inflation targeting and nominal interest rate rules

Consider the following model for output and inflation determination in a closed economy:

$$y_{t+1} = \theta y_t - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + u_{t+1}, \qquad 0 < \theta < 1, \quad \sigma > 0, \tag{1}$$

$$\pi_{t+1} = \pi_t + \kappa y_{t+1} + \eta_{t+1}, \qquad \kappa > 0, \tag{2}$$

where y_t is log of output in period t, i_t is the nominal interest rate (the monetary policy instrument), π_t is the inflation rate, u_t and η_t are independent, mean-zero, serially uncorrelated shocks. E_j is the rational expectations operator conditional on information up to and including period j. It is assumed that $\sigma > \kappa$.

(i) Discuss equations (1) and (2), with emphasis on the monetary transmission mechanism and the stability properties in absence of policy intervention (a verbal discussion is sufficient).

A The central points are that there are persistence in both the IS and Phillips curve. Moreover, monetary policy takes effect with a one-period lag (in accordance with VAR evidence). The effect of demand changes are contemporaneous on inflation, however. In absence of policy intervention, the model is unstable, as, e.g., a positive demand shock will increase demand and inflation, subsequently lower the real interest rate, and then further expand future output and inflation, and so on.

The objective of the central bank is to conduct monetary policy so as to maximize

$$U = -\frac{1}{2}\mathrm{E}_t \sum_{j=1}^{\infty} \beta^j \pi_{t+j}^2, \qquad 0 < \beta < 1.$$

(ii) Show that the optimal interest rate rule is

$$i_t = \left(1 + \frac{\sigma - \kappa}{\kappa}\right)\pi_t + \sigma\theta y_t.$$

[Hint: Treat $\mathrm{E}_t y_{t+1} \equiv y_{t+1} - u_{t+1}$ as the policy instrument, and solve the maximization problem by dynamic programming treating π_t as the state variable. Use (2) to make the maximization unconstrained, and use (1) subsequently to derive i_t.]

A Using the hint, the relevant value function becomes

$$\begin{aligned} v(\pi_t) &= \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t \left\{ -\frac{1}{2}(\pi_{t+1})^2 + \beta v(\pi_{t+1}) \qquad \text{s.t. } (2) \right\} \\ &= \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t \left\{ -\frac{1}{2}\left(\pi_t + \kappa y_{t+1} + \eta_{t+1}\right)^2 + \beta v\left(\pi_t + \kappa y_{t+1} + \eta_{t+1}\right) \right\}, \\ &= \max_{\mathrm{E}_t y_{t+1}} \mathrm{E}_t \left\{ \begin{array}{c} -\frac{1}{2}\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right)^2 \\ +\beta v\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) \end{array} \right\}. \end{aligned}$$

The first-order condition is:

$$\begin{aligned} & -\mathrm{E}_t\kappa\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) \\ & +\mathrm{E}_t\beta\kappa v'\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) \\ = {} & 0, \end{aligned}$$

$$-\mathrm{E}_t\kappa\left(\pi_t + \kappa\mathrm{E}_t y_{t+1}\right) + \mathrm{E}_t\beta\kappa v'\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) = 0,$$

$$-\left(\pi_t + \kappa\mathrm{E}_t y_{t+1}\right) + \mathrm{E}_t\beta v'\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right) = 0.$$

Using the Envelope Theorem one gets**:**

$$\begin{aligned} v'(\pi_t) &= \mathrm{E}_t\left\{-\left(\pi_t + \kappa y_{t+1} + \eta_{t+1}\right) + \beta v'\left(\pi_t + \kappa y_{t+1} + \eta_{t+1}\right)\right\} \\ &= -\left(\pi_t + \kappa\mathrm{E}_t y_{t+1}\right) + \mathrm{E}_t\beta v'\left(\pi_t + \kappa\left[\mathrm{E}_t y_{t+1} + u_{t+1}\right] + \eta_{t+1}\right). \end{aligned}$$

So, it follows that $v'(\pi_t) = 0$. Hence, from the first-order condition,

$$\mathrm{E}_t y_{t+1} = -\frac{1}{\kappa}\pi_t.$$

To find the interest-rate rule, we use that we have from (1) and (2) that

$$\begin{aligned}
\mathrm{E}_t y_{t+1} &= \theta y_t - \sigma^{-1}\left(i_t - \mathrm{E}_t\left[\pi_t + \kappa y_{t+1} + \eta_{t+1}\right]\right), \\
\mathrm{E}_t y_{t+1}\left(1 - \sigma^{-1}\kappa\right) &= \theta y_t - \sigma^{-1}\left(i_t - \pi_t\right), \\
\mathrm{E}_t y_{t+1} &= \frac{\theta}{1 - \sigma^{-1}\kappa} y_t - \frac{\sigma^{-1}}{1 - \sigma^{-1}\kappa}\left(i_t - \pi_t\right)
\end{aligned}$$

So, inserting $\mathrm{E}_t y_{t+1} = -\kappa^{-1}\pi_t$, the interest-rate rule follows from

$$-\frac{1}{\kappa}\pi_t = \frac{\theta}{1 - \sigma^{-1}\kappa} y_t - \frac{\sigma^{-1}}{1 - \sigma^{-1}\kappa}\left(i_t - \pi_t\right),$$

$$-\frac{\sigma - \kappa}{\kappa}\pi_t = \sigma\theta y_t - \left(i_t - \pi_t\right),$$

$$i_t = \sigma\theta y_t + \left[1 + \frac{\sigma - \kappa}{\kappa}\right]\pi_t,$$

which was what we should show.

(iii) Comment on the coefficient on π_t in the interest-rate rule, with special emphasis on how it affects the stability properties of the model. Also, discuss how the coefficients on π_t and y_t depend on the underlying parameters of the model, and evaluate whether the parameters reveal anything about the "strict" inflation-targeting preferences of the central bank.

A The main issue is that the coefficient is greater than one (as $\sigma > \kappa$). Hence, it is an active Taylor-type rule, such that any rise in inflation is met by a larger increase in the nominal interest rate. This increases the real interest rate, and will serve to contract output and thus reduce inflation. Hence, it serves a stabilizing role in case of a, e.g., positive demand shock mentioned in (i).
Moreover, it can be seen that σ (κ) increases (reduces) the inflation coefficient. This is because when σ increases or κ decreases, a larger nominal interest rate response is needed to stabilize inflation (as demand is less sensitive to interest-rate changes, and inflation is less sensitive to demand). Furthermore, it is observed that output changes will lead to nominal interest rate changes, even though the central bank is conducting "strict inflation targeting." The reason is that output changes provide information about inflation one period ahead (as long as there is output inertia; i.e., when $\theta > 0$). I.e., it serves as an intermediate target. Hence, the parameter values, and the variables in the interest rate rule, tell nothing about the preferences of the central bank.

QUESTION 3:

Monetary policy trade offs?

Consider the following log-linear model of a closed economy:

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(\widehat{i}_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right), \qquad \sigma > 0, & (1)\\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \qquad 0 < \beta < 1, \quad \kappa > 0, & (2)\\ \widehat{i}_t &= \phi \pi_t, \qquad \phi > 1, & (3) \end{aligned}$$

where x_t is the output gap (output's deviation from the flexible-price output), $\widehat{i}_t$ is the nominal interest rate's deviation from steady state, π_t is goods-price inflation and r_t^n is the natural rate of interest, which is assumed to be a mean-zero, i.i.d. shock. E_t is the rational-expectations operator conditional upon all information up to and including period t.

(i) Discuss (1) and (2) with focus on the underlying micro-economic foundations. What does (3) represent? Explain.

A Here is should be mentioned that (1), the dynamic IS curve, is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, $\widehat{i}_t -$ $\mathrm{E}_t \{\pi_{t+1}\}$, make consumers increase future consumption relative to current. The natural rate of interest is the real interest rate under flexible prices, which is consistent with (1) for $x_t = 0$. Equation (2), the New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be in effect for some periods. Equation (3) is a simple specification for how monetary policy, in terms of nominal interest rate setting, is determined. It is a simple Taylor-type rule where the nominal interest rate in increased (more than one-for-one) when inflation increases, which secures uniqueness of equilibrium in the model.

(ii) Derive the solutions for x_t and π_t. [Hint: Conjecture that the solutions are linear functions of r_t^n, and use the method of undetermined coefficients.] Comment on the role of the policy parameter ϕ in terms of the output gap and inflation's dependence on r_t^n, and discuss whether the parameter can be chosen such that the output gap and inflation are stabilized completely.

A We conjecture

$$x_t = ar_t^n, \qquad \pi_t = cr_t^n,$$

which due to the assumptions about r_t^n implies that

$$\mathrm{E}_t x_{t+1} = 0, \qquad \mathrm{E}_t \pi_{t+1} = 0.$$

Inserting the conjectures and their expected versions, along with the interest-rate rule, into (1) and (2), gives

$$\begin{aligned} ar_t^n &= -\sigma^{-1}\left(\phi c r_t^n - r_t^n\right) \\ cr_t^n &= \kappa a r_t^n. \end{aligned}$$

As this holds for all r_t^n, we obtain

$$\begin{aligned} a &= -\sigma^{-1}\phi c + \sigma^{-1}, \\ c &= \kappa a. \end{aligned}$$

We then get

$$\begin{aligned} a &= -\sigma^{-1}\phi\kappa a + \sigma^{-1}, \\ a &= \frac{\sigma^{-1}}{1+\sigma^{-1}\phi\kappa}, \end{aligned}$$

and thus

$$c = \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\phi\kappa}.$$

We can the combine the identified parameters with the conjecture and present the solutions for the output gap and inflation:

$$\begin{aligned} x_t &= \frac{\sigma^{-1}}{1+\sigma^{-1}\phi\kappa} r_t^n, \\ \pi_t &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\phi\kappa} r_t^n. \end{aligned}$$

First, one observe that if ϕ approaches infinity, the impact of the shock to the natural rate of interest will be fully neutralized on both the output gap and inflation. This is due to the "divine coincidence" whereby stabilizing one variable is consistent with stabilizing the other. Here, e.g., responding sufficiently aggressively to inflation will neutralize the impact of the shock on the output gap, which through the Phillips curve will imply price stability. Hence, it is possible to attain full stabilization of inflation and the output gap through monetary policy by setting $\phi = \infty$.

Assume that a welfare-relevant loss function can be written as

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\pi_t^2 + \lambda x_t^2 \right), \qquad \lambda > 0. \tag{4}$$

(iii) Derive the welfare-optimal values of x_t and π_t under discretionary policymaking [hence, equation (3) no longer applies]. For this purpose, treat x_t as the policy instrument, and show that the relevant first-order condition for optimal policy together with (2) yield the difference equation

$$\pi_t = \frac{\lambda \beta}{\kappa^2 + \lambda} \mathrm{E}_t \pi_{t+1}, \tag{5}$$

and that optimal inflation is therefore uniquely given by

$$\pi_t = 0.$$

Discuss why variations in the natural rate of interest do not appear in this solution, and discuss whether commitment of the central bank can improve on policymaking.

A Under discretion, expectations are taken as given, so we have a sequence of one-period minimization problems:

$$\begin{aligned} \min_{x_t} L(\pi_t, x_t) &\equiv \frac{1}{2} \left(\pi_t^2 + \lambda x_t^2 \right) \\ \text{s.t.} \quad \pi_t &= \kappa x_t + v_t \\ v_t &\equiv \beta \mathrm{E}_t \pi_{t+1} \end{aligned}$$

The relevant first-order condition is

$$\lambda x_t = -\kappa \pi_t.$$

Insert this back into (2) to eliminate x_t:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \frac{\kappa^2}{\lambda} \pi_t,$$

which is rewritten as

$$\begin{aligned} \pi_t \left(1 + \kappa^2 / \lambda \right) &= \beta \mathrm{E}_t \pi_{t+1}, \\ \pi_t \left(\lambda + \kappa^2 \right) &= \lambda \beta \mathrm{E}_t \pi_{t+1}, \end{aligned}$$

which can readily be written as (5).
Since, $\lambda\beta/\left(\lambda+\kappa^2\right) < 1$, (5) has a unique stationary solution. Clearly, this solution is $\pi_t = 0$. The output gap follows by using the first-order condition as

$$\begin{aligned} x_t &= -\frac{\kappa}{\lambda}\pi_t, \\ &= 0. \end{aligned}$$

It is seen that optimal policy under discretion implies full stabilization of inflation and the output gap. The shock to the natural rate of interest does not pose a policy trade off, and its impact on the output gap and inflation can be fully offset by an identical change in the nominal interest rate. Hence, in equilibrium, $\widehat{i}_t = r_t^n$. Since there are no policy trade-off, monetary policy attains the first best in this model under discretion. Hence, the discretion and commitment solutions coincide, and nothing would be gained by commitment in this case.